

Solar neutrinos: Oscillations or No-oscillations?

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The Nobel prize in physics 2015 has been awarded “... for the discovery of neutrino oscillations which show that neutrinos have mass”. While SuperKamiokande (SK), indeed, has discovered oscillations, SNO observed effect of the adiabatic (almost non-oscillatory) flavor conversion of neutrinos in the matter of the Sun. Oscillations are irrelevant for solar neutrinos apart from small ν_e regeneration inside the Earth. Both oscillations and adiabatic conversion do not imply masses uniquely and further studies were required to show that non-zero neutrino masses are behind the SNO results. Phenomena of oscillations (phase effect) and adiabatic conversion (the MSW effect driven by the change of mixing in matter) are described in pedagogical way.

I. INTRODUCTION

The Nobel prize in physics 2015 has been awarded to T. Kajita, Super-Kamiokande, and A. B. McDonald, Sudbury Neutrino Observatory (SNO). Super-Kamiokande (SK, among other things) studied properties of the atmospheric neutrinos. The oscillatory dependence of the number of μ -like events on L/E (distance over energy) has been observed which is the key signature of oscillations [1]. The SNO collaboration studied the solar neutrinos: the flux of the ν_e neutrinos and the total flux of all neutrino flavors (ν_e, ν_μ, ν_τ) have been measured via the charged current interactions and the neutral current interactions correspondingly. Comparing the two fluxes, SNO has established transformation of ν_e into ν_μ and ν_τ [2].

The prize has been awarded “... for the discovery of neutrino oscillations, which shows that neutrinos have mass”. Two remarks concerning this citation are in order

- While SK has, indeed, discovered neutrino oscillations, the SNO has established, as we understood later, almost *non-oscillatory* adiabatic flavor conversion (the MSW effect). Oscillations are irrelevant for interpretation of the SNO results apart from small regeneration effect inside the Earth.
- Oscillations do not necessarily imply the mass.

In what follows I will explain these two points and give simple description of the phenomena of oscillations and adiabatic conversion. Comments on physics and terminology will be given in conclusion ¹.

Neutrino oscillations and adiabatic conversion are consequences of mixing [3], [4]. Graphic representation of the vacuum mixing is shown in Fig. 1. There are three types of neutrinos: ν_e, ν_μ, ν_τ which we refer to as neutrinos with definite flavors. Mixing means that the flavor neutrino states do not coincide with the mass states ν_1, ν_2, ν_3 . The flavor states are combinations

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¹ This paper is based on several colloquia delivered by the author during the last year.

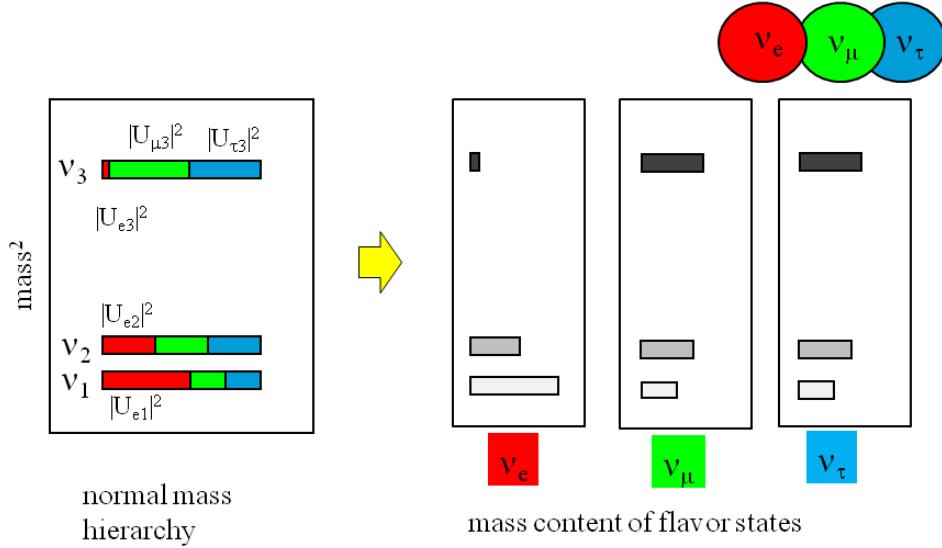


FIG. 1. Graphic representation of neutrino mixing. *Left panel:* neutrino mass spectrum and flavor composition of the mass eigenstates. The mass states are shown by boxes. Each box contains mixture of different flavors (color parts). Areas of colored parts give probabilities to find the corresponding flavor neutrino in a given mass state, if the area of the box is 1. *Right panel:* Mass composition of the flavor states. The gray-black boxes correspond to the mass states in a given flavor state. Relative areas of the boxes give probabilities to find the corresponding mass state in a given flavor state.

(mixture) of mass states, and inversely, the mass states are combinations of the flavor states (see Fig. 1, left). According to this Figure, e.g. ν_2 is composed of nearly equal amount of ν_e , ν_μ , ν_τ . In ν_3 the flavor states ν_μ and ν_τ are presented almost equally with very small admixture of ν_e . Therefore, ν_3 would show up (interact) with probability ~ 0.48 as ν_μ , with probability ~ 0.5 as ν_τ and with probability 0.02 as ν_e . If the beam of high energy ν_3 is created, it will produce (in the CC interactions) numbers of e , μ and τ leptons with fractions 2 : 48 : 50.

Second aspect of mixing is that the flavor states are combinations of the mass states (Fig. 1, right). E.g. ν_e is composed of about 2/3 of ν_1 , 1/6 of ν_2 and 1/6 of ν_3 . A mass spectrometer studying ν_μ (in a “gedadken” experiment) would find three peaks: at values of mass m_1 , m_2 and m_3 with intensities 2/3 : 1/6 : 1/6 correspondingly.

The key point which can not be seen in this figure is that flavor states are coherent combinations of the mass states. The mass states ν_i , in a given flavor state ν_α ($\alpha = e, \mu, \tau$) have definite relative phases.

In terms of Fig. 1 left, SK has measured value of large (2-3) mass splitting and distribution of the ν_μ and ν_τ flavors (green and blue) in the third mass state ν_3 . SNO has constrained the small (1-2) mass splitting and established the distribution of the ν_e flavor (red) in ν_1 and ν_2 .

II. OSCILLATIONS

Neutrino oscillations [3] [5] are

- consequence of mixing: production and propagation of mixed states;
- manifestation of interference;

- effect of change of the relative phase: increase with time and distance of the phase difference between the eigenstates of the Hamiltonian which compose a propagating mixed state. It is this increase of the phase that changes the interference effect.

For simplicity we will consider two-neutrino mixing: ν_μ and ν_τ keeping in mind application to the SK results on the atmospheric neutrinos:

$$\begin{aligned}\nu_\mu &= \cos \theta_{23} \nu_2 + \sin \theta_{23} \nu_3, \\ \nu_\tau &= -\sin \theta_{23} \nu_2 + \cos \theta_{23} \nu_3 = e^{i\pi} \sin \theta_{23} \nu_2 + \cos \theta_{23} \nu_3.\end{aligned}\quad (1)$$

Eq. (1) means that ν_μ , the neutrino state produced with muon e.g., in pion decay, is certain coherent combination of two states with definite masses. The combination is characterized by weights given by $\cos \theta_{23}$ and $\sin \theta_{23}$ and the relative phase. According to equation (1) there is an additional “intrinsic” phase $\phi_{int}^\tau = \pi$ in the ν_τ state, while $\phi_{int}^\mu = 0$ in ν_μ ². Coherence means that the relative phases between ν_2 and ν_3 in ν_μ and ν_τ are not random or averaged, and they determine properties (in particular interactions) of these states.

The equations (1) can be inverted:

$$\begin{aligned}\nu_2 &= \cos \theta_{23} \nu_\mu - \sin \theta_{23} \nu_\tau, \\ \nu_3 &= \sin \theta_{23} \nu_\mu + \cos \theta_{23} \nu_\tau,\end{aligned}\quad (2)$$

which shows that the mass eigenstates are combinations of the flavor states.

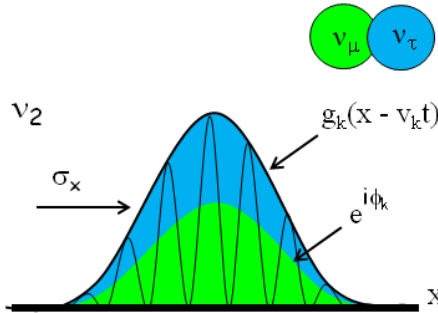


FIG. 2. Wave packet of the mass state ν_2 . The oscillatory pattern is inscribed in the envelope which moves with group velocity. The green and blue parts show the flavor composition of the mass state.

Propagation of the mass states is described by wave packets³. The wave packet for the k -eigenstate ($k = 2, 3$) in the configuration space (t, x) can be parametrized as the product of the shape and phase factors [6]:

$$\psi_k = g_k(x - v_k t) e^{i\phi_k}. \quad (3)$$

The shape factor g_k depends on combination of space-time coordinates $(x - v_k t)$, where v_k is the group velocity, and therefore describes propagation of the packet. g_k is an envelope of the

² This additional phase is related to orthogonality of the two flavor states. For definiteness we put it on ν_τ . We could put it on ν_μ or, in general, introduce the phases in both states but the difference should be π .

³ Although for practical purpose the wave packet description can be avoided in most of the cases, it is necessary to clarify the conceptual issues.

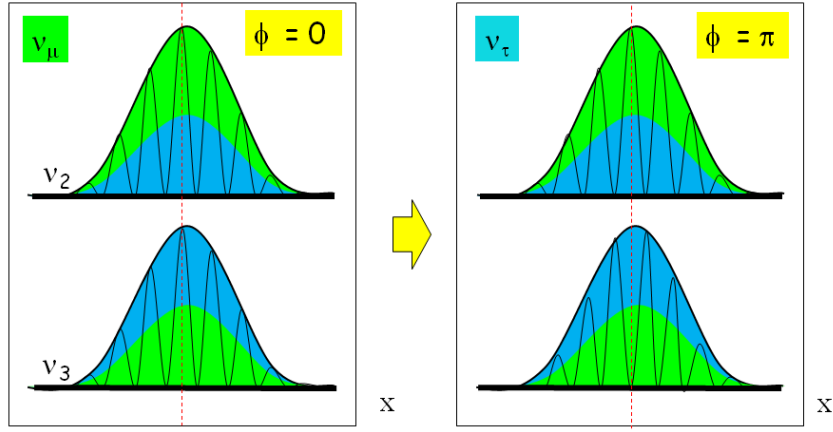


FIG. 3. Wave packet picture of neutrino oscillations $\nu_\mu \rightarrow \nu_\tau$. *Left panel:* picture of muon neutrino. *Right panel:* tau neutrino. The two states distinguished mainly by the phase difference (shift of the oscillatory patterns.) Each WP has muon (green) and tau (blue) parts according to Eq. (2).

packet. The phase factor, $e^{i\phi_k}$, produces an oscillatory pattern inscribed in the envelop (see Fig. 2). Here

$$\phi_k(x, t) = p_k x - E_k t \quad (4)$$

is the phase with p_k and E_k being the mean momentum and mean energy in the packet (see Fig. 2). The size of the WP and its shape are determined by the production process: by kinematics of the elementary reaction and localization of parents (source) of neutrinos.

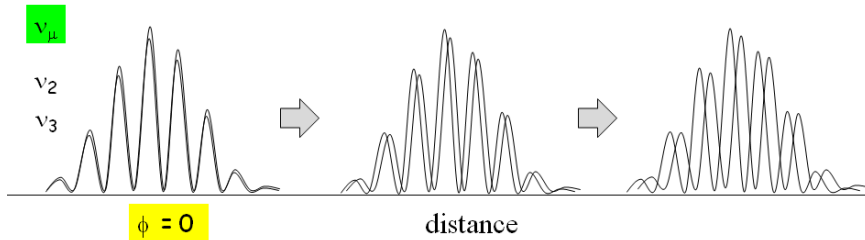


FIG. 4. Wave packet picture of neutrino oscillations: shift of the phase between oscillatory factors.

In the 2ν -approximation propagation in vacuum of the state produced as ν_μ (1) is described by two wave packets which correspond to mass states ν_2 and ν_3 (and similarly – for ν_τ), see Fig. 3. In this picture there are several simplifications which are not essential for physics of oscillations. (i) We show one-dimensional (1D) picture; in 3D the WP may have spherical front. (ii) For visibility we show one packet under another but in reality they overlap in space, (iii) we show only the upper parts of the packets. In reality the packet is symmetric with respect to the propagation axis (and actually, it occupies a complex plane). Parts of different WP with the same flavor (color in Fig. 3) interfere and the result of interference depends on the phase difference. We assume that the WP are short enough, so that the phase difference is the same along the whole packet (from front to back part).

In the course of propagation additional phase difference between the mass eigenstates appears: Due to difference of masses the states ν_2 and ν_3 have different *phase velocities*,

$v_k^{ph} = E_k/p_k$ ($k = 2, 3$). The latter leads to appearance of the phase difference (shift of the oscillatory patterns, see Fig. 4) during propagation:

$$\phi_{osc} \equiv \phi_3 - \phi_2 \approx \frac{\Delta m_{32}^2 L}{2E} \quad (5)$$

which we call the oscillation phase. The total phase difference between mass states in the ν_τ is

$$\phi_\tau = \phi_{int}^\tau + \phi_{osc} = \pi + \phi_{osc}. \quad (6)$$

Since $\phi_{int}^\mu = 0$, the phase in the ν_μ state equals

$$\phi_\mu = \phi_{osc}. \quad (7)$$

Suppose ν_μ is produced at $t \approx 0$, $x \approx 0$ (actually we can not indicate exact time and space point due to finite size of the source localization region related to the uncertainty principle.) In this moment the oscillation phase $\phi_{osc} = 0$, so that $\phi_\mu = 0$ and $\phi_\tau = \pi$. Consequently, there is a constructive interference of the muon parts and destructive interference of the tau parts (see Fig. 3, left). In fact, having the same amplitudes (see Appendix A), the tau parts cancel completely, as it should be in the ν_μ state.

Increase of the phase difference with distance and time will change the interference picture, see Fig. 4. For $\phi_{osc} \neq 0$ the tau parts will not cancel completely which means that ν_τ component appears in the neutrino state originally produced as ν_μ . When $\phi_{osc} = \pi$ (corresponding to the strongest deviation from the initial state) we have $\phi_\mu = \pi$ and $\phi_\tau = 2\pi$. This leads to the destructive interference of the muon parts of two packets and constructive interference of the tau parts (Fig. 3, right). Thus, the originally produced muon neutrino is converted partially or completely (if mixing is maximal) to the tau neutrino. The neutrino system returns back to the initial flavor state when $\phi_{osc} = 2\pi$. According to (5) the corresponding distance – the oscillation length – equals

$$l_\nu = L(2\pi) = \frac{4\pi E}{\Delta m^2}. \quad (8)$$

From Fig. 3 one can immediately obtain the $\nu_\mu \rightarrow \nu_\mu$ oscillation probability (see Appendix A):

$$P_{\mu\mu} = 1 - P_{\mu\tau} = 1 - \sin^2 2\theta_{23} \sin^2 \frac{\Delta m_{32}^2 L}{4E}, \quad (9)$$

where we used explicit expression for ϕ_{osc} (5). It is this phenomenon that happens in atmospheric neutrinos and was detected by Super-Kamiokande.

III. NO-OSCILLATIONS

A. SNO results and their interpretation

In the case of solar neutrinos we deal with mixing of ν_e and ν_a (the latter is certain combination of the muon and tau neutrinos):

$$\begin{aligned} \nu_e &= \cos \theta_{12} \nu_1 + \sin \theta_{12} \nu_2, \\ \nu_a &= -\sin \theta_{12} \nu_1 + \cos \theta_{12} \nu_2. \end{aligned} \quad (10)$$

Here θ_{12} is the 1-2 vacuum mixing angle, ν_1 and ν_2 are the eigenstates with mass squared splitting Δm_{21}^2 .

SNO has measured the flux of ν_e , Φ_e , detecting the charged current interactions and the total flux of all active neutrinos, Φ_{NC} , detecting the neutral current events. Since only ν_e are produced in the Sun, the ratio of the fluxes [2]:

$$P_{SNO} \equiv \frac{\Phi_e}{\Phi_{NC}} = 0.340 \pm 0.023 \begin{smallmatrix} +0.029 \\ -0.031 \end{smallmatrix}, \quad (11)$$

being smaller than 1 implies an appearance of the ν_μ , ν_τ fluxes in originally produced ν_e flux, that is $\nu_e \rightarrow \nu_\mu$, ν_τ transformation. The ratio P_{SNO} gives the survival probability of the electron neutrinos. It turns out that value in (11) is close to the value of the ν_e —survival probability of the so called *non-oscillatory transition* [7]:

$$P_{non-osc} = \sin^2 \theta_{12} = 0.31, \quad (12)$$

(see later). The difference of values in (11) and (12),

$$P_{SNO} - P_{non-osc} = 0.03,$$

is due to small averaged oscillation effect in the Sun and the ν_e *regeneration* in the matter of the Earth (see review [8]).

Interpretation of the SNO result is *the adiabatic flavor conversion* of ν_e in matter with varying density (the MSW effect) [7, 9–11]. Complete expression for the survival probability, which reproduces (11), is

$$P_{SNO} = \sin^2 \theta_{12} + \cos 2\theta_{12} \cos^2 \theta_{12}^{m0} + f_{reg}. \quad (13)$$

Here the first term is the contribution from the non-oscillatory transition. The second one is the effect of averaged oscillations in the Sun:

$$P_{osc} = \cos 2\theta_{12} \cos^2 \theta_{12}^{m0} \approx 0.015, \quad (14)$$

where θ_m^0 is the mixing angle in matter in the production point. Detailed study of this contribution and its dependence on energy has been performed in [12], [13]. Finally,

$$f_{reg} \approx 0.015 \quad (15)$$

is the so called ν_e —regeneration factor in the Earth. The regeneration is due to (averaged) oscillations of the mass states in matter which do not exist in vacuum.

Thus, the effect observed by SNO is mainly the non-oscillatory transition with less than 10% contribution from the averaged oscillations. Actually, there is no much sense to speak about oscillations: for the SNO energies the coherence between the eigenstates is lost at distances comparable with solar radius (see below).

It is not possible to obtain $P_{non-osc}$ from the oscillation formula. Recall that the averaged survival 2ν probability in vacuum is $P = 1 - 0.5 \sin^2 2\theta_{12} \approx 0.58$, and in view of smallness of the 1-3 mixing it can never be smaller than ≈ 0.5 . The reason is that physics (dynamics) is different. To explain this result let us recall some basics of the matter effects.

B. Refraction

At low energies inelastic interactions of neutrinos can be neglected. For these neutrinos the Earth, the Sun, other stars look like transparent balls of glass for the light. Elastic forward scattering produces the refraction [9] – dominant phenomena described by the refraction index n . The deviation of n from unity is very small. E.g. for 10 MeV neutrino energy: $|n-1| = 10^{-20}$ in the Earth and $|n-1| = 10^{-18}$ in the center of the Sun. So, what happens at low energies is extremely small reflection from sharp borders between layers with different densities and bending of trajectories in non-uniform medium with smooth density change.

The matter effect can be equivalently described by the potential V :

$$n - 1 = \frac{V}{p}. \quad (16)$$

Not potential itself but difference of potentials has physical meaning. For single neutrino species reflection and bending of trajectory are related to change of V in space. For system of two and more mixed neutrinos the difference of potentials can be realized in different way – due to the difference of interactions of two neutrinos [9]. For ν_e and ν_a the difference of potentials in usual medium equals $V = V_e - V_\mu = \sqrt{2}G_F n_e$. Here G_F is the Fermi coupling constant and n_e is the number density of electrons. The inverse of the potential, $l_0 \equiv 2\pi/V$, gives the refraction length which determines spatial scales of the phenomena.

Amazing fact is that the energies of neutrinos from natural sources (MeV - GeV range), the radius of the Earth R_E and Δm_{21}^2 (which comes from some new physics at very high mass scales) turn out to be related in such a way that

$$\frac{\Delta m_{21}^2}{2E} \sim V \sim 1/R_E. \quad (17)$$

For neutrinos from pion and muon decays in flight (\sim few hundreds MeV - GeV) one should use the 1-3 mass splitting Δm_{31}^2 instead of Δm_{21}^2 . It is this “conspiracy” of small quantities that is behind the fact that we can observe oscillations and various matter effects in spite of smallness of V .

C. Mixing in matter

In general, the flavor mixing is defined with respect to the eigenstates of the Hamiltonian (Fig. 5). In vacuum we have the Hamiltonian H_0 whose eigenstates are the mass states ν_{mass} . Mixing in vacuum connects the flavor states ν_f and ν_{mass} as in eq. (10). In matter, the total Hamiltonian differs from H_0 . We should add the potential V (the matrix of potentials), so that $H = H_0 + V$. Therefore the eigenstates of H , ν_m , differ from the eigenstates of H_0 : $\nu_m \neq \nu$. So, instead of (10) we have

$$\begin{aligned} \nu_e &= \cos \theta_{12}^m \nu_{1m} + \sin \theta_{12}^m \nu_{2m}, \\ \nu_a &= -\sin \theta_{12}^m \nu_{1m} + \cos \theta_{12}^m \nu_{2m}, \end{aligned} \quad (18)$$

and mixing angle in matter θ_m differs from mixing in vacuum: $\theta_m \neq \theta$. Let us reiterate: while the flavor states are the same both in vacuum and in matter, the eigenstates are different, and correspondingly, the mixing angles are different.

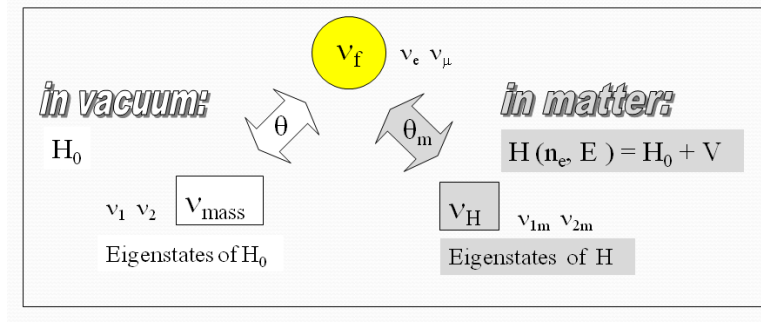


FIG. 5. Mixing in vacuum and in matter.

Eq. (18) can be inverted, expressing the eigenstates in terms of the flavor states:

$$\begin{aligned}\nu_{1m} &= \cos \theta_{12}^m \nu_e - \sin \theta_{12}^m \nu_a, \\ \nu_{2m} &= \sin \theta_{12}^m \nu_e + \cos \theta_{12}^m \nu_a.\end{aligned}\tag{19}$$

According to these equations mixing (by definition) determines flavor composition of the eigenstates. Mixing angle fixes (uniquely) the flavor of the eigenstates of the Hamiltonian. Mixing is equivalent to flavor composition. In matter mixing depends on density. When density changes the mixing changes and therefore flavor of eigenstates changes.

The vacuum mixing angle is the fundamental parameter of the Hamiltonian. In matter the mixing angle becomes a *dynamical variable*. Since $H_0 = H_0(E)$ and $V = V(n)$ we have

$$\theta_m = \theta_m(n(t), E).\tag{20}$$

The mixing angle is not a constant anymore and dependence of the angle on the density and energy leads to various new phenomena. (Introduction of notions of mixing and eigenstates in matter is very useful both for solution of the evolution equation and for understanding physics; further comments are in Appendix B.) Notice that also E may depend on time as is realized, e.g., due to the redshift in the Universe.

Dependence of $\sin^2 2\theta_m$ on energy or density has resonance character. In the resonance $\sin^2 2\theta_m = 1$ or

$$\theta_m(E, n) = \frac{\pi}{4},\tag{21}$$

and this is satisfied under the resonance condition:

$$V(n) = \frac{\Delta m_{21}^2}{2E} \cos 2\theta_{12}.\tag{22}$$

This condition determines the resonance density for a given energy E : $n_R = n_R(E)$, or resonance energy in medium with a given density n : $E_R = E_R(n)$. The resonance energy and resonance density determine scales of various phenomena.

In matter with constant density θ_m plays the same role as θ in vacuum⁴. In particular, $\sin^2 2\theta_m$ gives the the depth of oscillations. At the resonance energy the depth becomes maximal: $\sin^2 2\theta_m = 1$. This phenomenon was called *the resonance enhancement* of oscillations

⁴ By the way, θ can also be considered as the angle in medium (average of the classical field), keeping in mind that masses and mixing of neutrinos are generated by their interactions with the vacuum expectation value of the Higgs field(s).

[11].

In the case of the standard neutrino interactions the specific dependence of mixing on V is determined by the fact that the potential does not change flavor being “flavor-diagonal”. Scattering which produces this potential does not change flavor. So, in the Hamiltonian it appears in diagonal elements, while the flavor is changed by the off-diagonal term $\sin 2\theta \Delta m^2 / 4E$ which does not depend on density. Therefore at very high densities – much above the resonance $V \gg V(n_R)$, the diagonal elements dominate and therefore mixing is suppressed. This has two realizations depending on signs of Δm^2 and V (and V has opposite signs for neutrino and antineutrinos): $\theta_{12}^m \rightarrow \pi/2$ ($n \gg n_R$) in the resonance channel, where the condition (22) is satisfied, and $\theta_{12}^m \rightarrow 0$ which occurs in the non-resonance channel. In the resonance channel with decrease of density the angle θ_{12}^m decreases from $\pi/2$: it becomes $\pi/4$ in the resonance and then approaches the vacuum value $\theta_m \approx \theta$ when $n \ll n_R$.

D. Adiabatic conversion

Adiabatic conversion is realized in matter with slowly varying density [7, 10]. As we discussed in the previous section mixing connects uniquely the flavor state with the eigenstates. So, dynamics of flavor transformations is reduced to the dynamics of the eigenstates. Transitions between the eigenstates $\nu_{1m} \leftrightarrow \nu_{2m}$ are governed by the adiabaticity condition which involves density gradient on the way of neutrinos. If density changes slowly enough this condition is fulfilled (see Appendix C), and consequently, transitions between the eigenstates are negligible. The absence of the

$$\nu_{2m} \leftrightarrow \nu_{1m} \quad (23)$$

transitions is the essence of the adiabatic propagation.

Let us describe now the non-oscillatory transitions. The benchmark point here is again the resonance density n_R . Suppose the electron neutrinos are produced at very high densities $n \gg n_R$. As discussed in sect. IIIC, in this case mixing is very strongly suppressed and in the resonance channel $\theta_{12}^{m0} \approx \pi/2$. This means, according to eq. (18), that the electron neutrino is essentially composed of single eigenstate $\nu_e \approx \nu_{2m}$. In turn, as follows from (19) this eigenstate has mainly the electron flavor: $\nu_{2m} \approx \nu_e$. The admixture of the second eigenstate is strongly suppressed and can be neglected (see Fig. 6a). If single eigenstate ν_{2m} propagates, there is no interference since simply there nothing to interfere with. Consequently, there is no oscillations in principle. The phase of this state is irrelevant (only the difference of phases has physical meaning).

If the density changes on the way of neutrino, then mixing θ_{12}^m changes, and consequently, flavor of the eigenstate ν_{2m} changes. With decrease of density the ν_a component in ν_{2m} increases. In resonance ν_{2m} has equal fraction of ν_e and ν_a (see Fig. 6d), and then fraction of ν_a becomes bigger than the one of ν_e approaching the vacuum value. At low density, the flavor of the eigenstate is determined by the vacuum mixing (Fig. 6c).

When density changes slowly (adiabatically) another eigenstate ν_{1m} is not produced. So, during whole the evolution $\nu \approx \nu_{2m}$, and consequently, change of flavor of the whole state follows the flavor of ν_{2m} and the latter, in turn, follows the density variations. This is realization of the adiabatic non-oscillatory regime:

$$\nu_e \approx \nu_{2m} \rightarrow \nu_2. \quad (24)$$

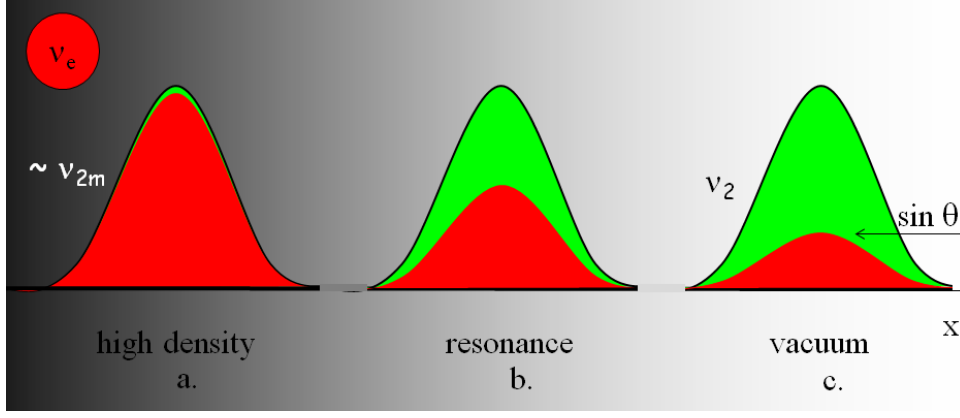


FIG. 6. Wave packet picture of the non-oscillatory adiabatic conversion of the electron neutrino. Initial density $n \gg n_R$. Shown are snapshots of the propagating neutrino state for three different densities. The neutrino state ν_e consists essentially of a single eigenstate ν_{2m} . The flavor of this eigenstate determined by mixing angle changes according to the density change. The irrelevant oscillatory pattern (as in Figs. 2, 3) is not shown here.

Now we can obtain the probability of the transition (survival probability) immediately:

$$P_{ee} \approx |\langle \nu_e | \nu_2 \rangle|^2 = \sin^2 \theta_{12}. \quad (25)$$

If initial density is not very large, then the produced ν_e is composed of two eigenstates – the admixture of ν_{1m} can not be neglected, see Fig. 7a. The admixtures are determined by mixing in the initial moment (production point). In this case apart from change of flavors of individual eigenstates ν_{1m} and ν_{2m} also interference takes place and so an interplay of the adiabatic conversion and oscillations should be observed.

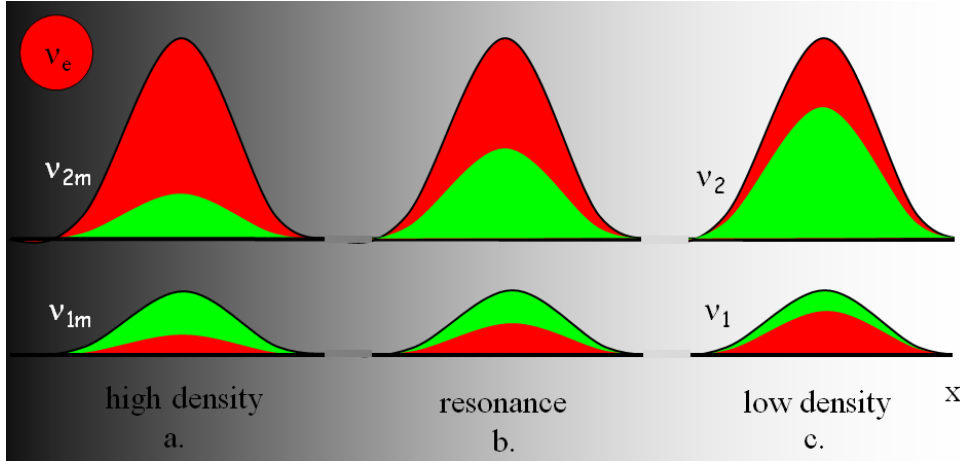


FIG. 7. Wave packet picture of the adiabatic conversion of the electron neutrino in general case. Sizes of the WP do not change; flavors of the eigenstates change according to density change. Interference of the same flavor parts takes place. The oscillatory pattern is not shown: the wave packets of ν_1 and ν_2 eventually separate and the interference terminates.

Again, if the density changes slowly the transitions $\nu_{2m} \leftrightarrow \nu_{1m}$ are suppressed, and therefore the amplitudes (shape factors) of the wave packets of ν_{2m} and ν_{1m} do not change. Flavors of the eigenstates being determined by mixing angle θ_{12}^m follow the density change. The

dependence of the survival probability on distance for the adiabatic conversion is shown in the lower panel of Fig. 8, being compared with spatial picture of oscillations (the upper panel).

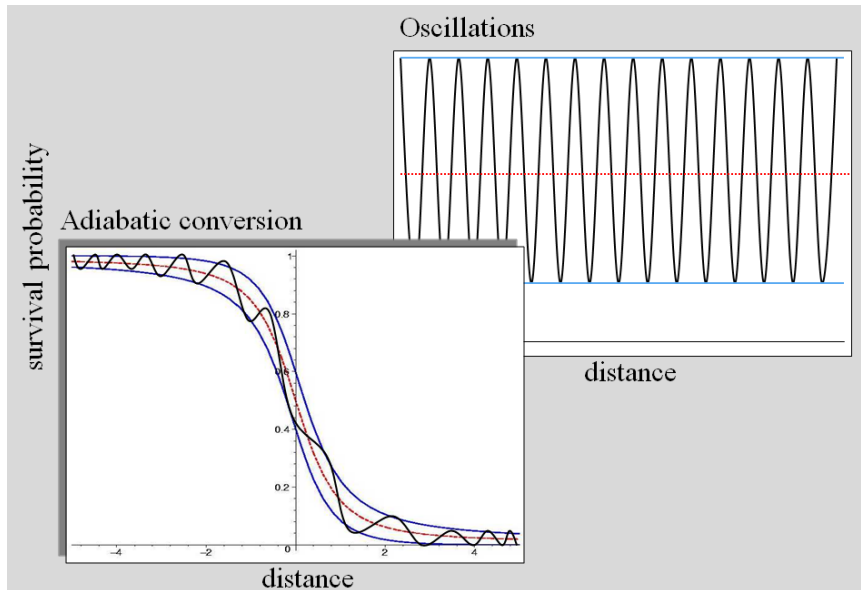


FIG. 8. Spatial picture of oscillations (upper panel) and adiabatic conversion (lower panel). Shown are the dependence of the survival probabilities on distance.

Even in the presence of both eigenstates for solar neutrinos the oscillations are irrelevant. Indeed, in the configuration space loss of the propagation coherence occurs: due to difference of the group velocities (they should be computed in matter) the wave packets which correspond to two eigenstates shift with respect to each other and eventually separate (Fig. 9). The difference of group velocities in vacuum is

$$\Delta v_{gr} = \frac{\Delta m^2}{2E^2}. \quad (26)$$

The shift equals $\Delta v_{gr}L$. Therefore complete separation of the packets occurs at the distance L_{coh} (coherence length) determined by

$$\Delta v_{gr}L_{coh} = \frac{\Delta m^2 L}{2E^2} = \sigma_x, \quad (27)$$

where σ_x is the size of the wave packet. For solar neutrinos depending on energy L_{coh} varies from few hundreds of kilometers to several solar radius's⁵.

Conversion of the solar neutrinos can be viewed as the incoherent production and propagation of the eigenstates in matter, and change of flavors of these eigenstates in course of propagation according to density change. Admixtures (weights) of the eigenstates in a given propagating state do not change being determined by θ_{12}^{m0} – the mixing angle in matter in the production point. The accuracy of this description is $\sim 10^{-4}$.

⁵ This loss of coherence can not be restored at the detection, since it would require too long coherence time of detection, or equivalently, unachievable energy resolution.

The survival probability can be obtained from the Fig. 7 immediately. The amplitudes of the packets in initial moment, and consequently, in any other moment (due to adiabaticity) are determined by mixing in the initial state: $\cos \theta_{12}^{m0}$ for ν_{1m} and $\sin \theta_{12}^{m0}$ for ν_{2m} . Due to adiabaticity and loss of coherence the two wave packets evolve independently and effects of these two packets should sum up in the probability. The contribution to the probability to find ν_e from ν_{1m} which transforms to ν_1 is $(\cos \theta_{12} \cos \theta_{12}^{m0})^2$ where $\cos \theta_{12}$ gives fraction of ν_e in ν_1 . Similarly, the contribution from ν_{2m} is $(\sin \theta_{12} \sin \theta_{12}^{m0})^2$. The sum of the contributions reproduces the first and second terms in Eq. (13).

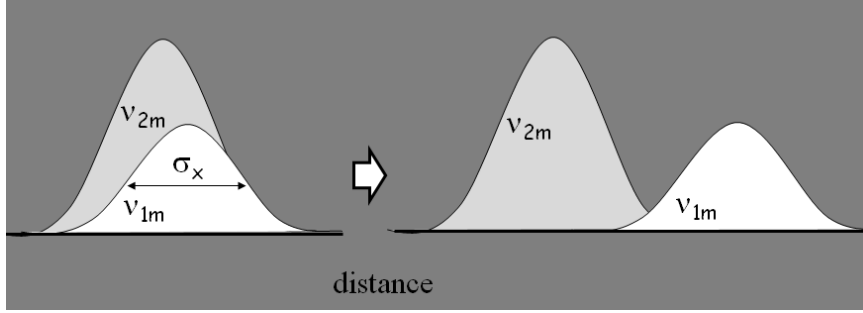


FIG. 9. Loss of propagation coherence due to spatial separation of the wave packets.

Let us mention that there is the third effect of propagation: spread of individual wave packets in space. The spread is related to the presence of different momenta in a given wave packet: so that parts of the WP with higher energy will propagate faster (see Fig. 10, and details in [14]). It can be shown that spread of the WP does not change the coherence condition.

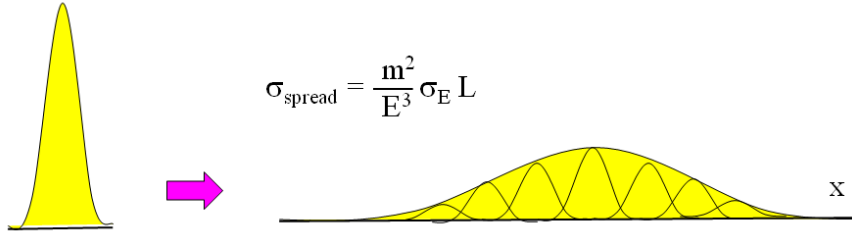


FIG. 10. Spread of the individual wave packet of the eigenstate with mass m . Different parts of the expanded WP of the size σ_x become effectively incoherent.

Bottom line: SNO had observed effect of the adiabatic conversion and loss of coherence.

E. Oscillations versus adiabatic conversion

Let us summarize the differences between the oscillations and adiabatic conversion. Neutrino oscillations are manifestation of interference which changes in space/time. The interference is determined by the phase difference between the two (or more) eigenstates of the Hamiltonian.

In the case of the non-oscillatory transition a produced neutrino state consists of a single eigenstate. Consequently, there is no interference, no phase difference, and no oscillations.

In pure form oscillations occur in vacuum and matter with constant density. In contrast, the adiabatic flavor conversion is the effect of propagation of neutrino in medium with slowly changing density.

Two different degrees of freedom are involved in oscillations and conversion:

- the phase ϕ_{osc} – in the case of oscillations. It changes the interference picture. The phase ϕ_{osc} is the key dynamical variable; mixing does not change.
- mixing angle θ_m – in the case of adiabatic conversion. Here the flavor of a neutrino state determined by $\theta_m(n)$ follows the density change on the way of neutrinos. The phase is irrelevant. The process is not periodic and irreversible for solar neutrinos.

Non-oscillatory transition is extreme case of the adiabatic conversion when initial density is much larger than the resonance density. This is realized approximately for high energy part of the solar neutrino spectrum.

Periodic (or quasi-periodic) L/E dependence of the probability is the main signature of oscillations. Oscillations show up as oscillatory dependence of the flavor of neutrino state as function of distance L or L/E , in general. Adiabatic conversion is independent of spatial scales - the corresponding probabilities do not depend on distance.

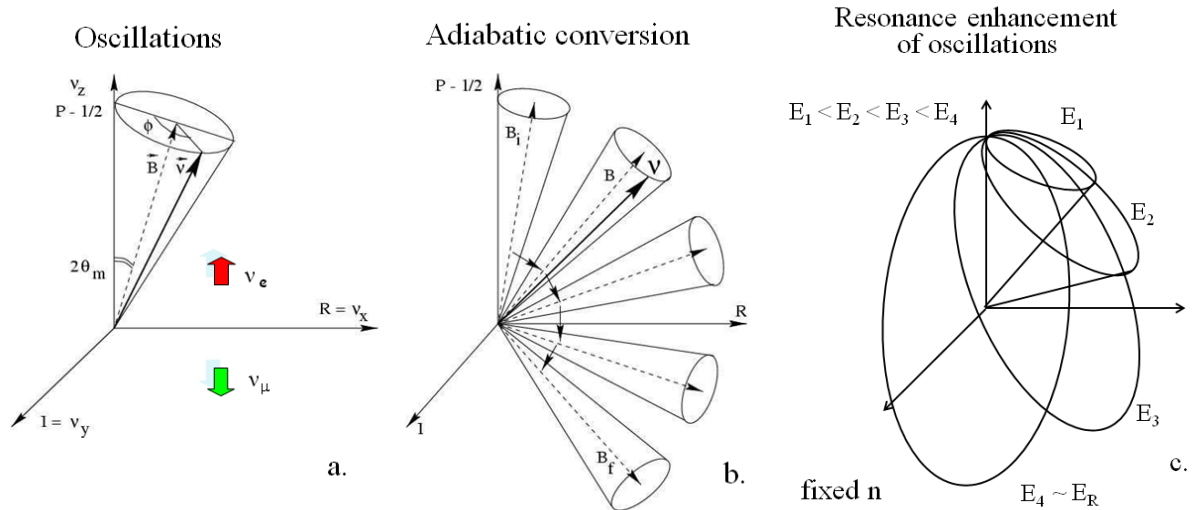


FIG. 11. Graphic representation of the neutrino oscillations (a), adiabatic conversion (b), and the resonance enhancement of oscillations (c). The non-oscillatory transition corresponds to b) with very small cone opening angle, so that the cone essentially coincides with the axis. In the case c) for different energies neutrino vector moves on the surface of the cone with different directions of the axis and cone angle. Shown are cones of rotation for different energies.

In Fig. 11 we show graphic representations of the neutrino oscillations and adiabatic conversion which are based on analogy with the electron spin precession in the magnetic field. Neutrino polarization vector in flavor space (“spin”) is moving in the flavor space around the “eigenstate axis” (magnetic field) whose direction is determined by the mixing angle $2\theta_m$. Oscillations are equivalent to the precession of the neutrino polarization vector around fixed axis, Fig. 11a. Oscillation probability is determined by projection of the neutrino vector on the axis z . The direction up of the neutrino vector corresponds to the ν_e , direction down – to ν_μ . Adiabatic conversion is driven by rotation of the cone itself, *i.e.* change of direction

of the magnetic field (cone axis) according to change of the mixing angle, Fig. 11b. Due to adiabaticity the cone opening angle does not change and therefore the neutrino vector follow rotation of axis.

IV. "... WHICH SHOWS THAT NEUTRINOS HAVE MASS"

A. SNO, oscillations and KamLAND

SNO has established the transformations of ν_e to ν_μ, ν_τ . Within the experimental error-bars the effects does not depend on energy. No L/E dependence has been observed and mechanism of transformation has not been identified. After SNO publications a number of solutions of the solar neutrino problem still existed: LMA MSW and LOW MSW conversion, resonant spin-flavor precession, Lorentz symmetry violation, decoherence, neutrino decay, *etc.* Very good description of the data has been provided by the so called non-standard neutrino interactions with zero neutrino masses.

It is the KamLAND (Kamioka Large Antineutrino detector) [15] that has selected unique solution of the solar neutrino problem (in assumption of the CPT invariance) and showed that non-zero neutrino mass is behind the SNO result. KamLAND studied the antineutrino fluxes from many atomic reactors with average baseline about 180 km. The L/E dependence of the survival probability has been observed. Extracted values of Δm_{21}^2 and θ_{12} were in agreement with LMA MSW solution.

Solar neutrino experiments and KamLAND have completely different environments: Solar neutrinos propagate in matter with varying density, then in vacuum and finally in matter of the Earth. In KamLAND we deal essentially with oscillations in vacuum (matter effect is present but very small). Coincidence of the parameters $\Delta m_{21}^2, \theta_{12}$ determined in solar experiments and KamLAND had a number of implications:

- confirmation of CPT invariance;
- correctness of theory of neutrino oscillations in vacuum and in matter, and of adiabatic conversion;
- selection of unique solution of the solar neutrino problem,
- strong indication that neutrino mass is behind oscillations and adiabatic conversion.

15 years after with more data collected we see some difference of Δm_{21}^2 extracted from the solar data and KamLAND. The significance of this difference is slightly bigger than 2σ . It can be just statistical fluctuation or some systematics, but may turn out to be effect of new physics.

B. Oscillations and mass

Oscillations do not need the mass. Recall that it was the subject of the classical Wolfenstein's paper [9] to show that oscillations can proceed for massless neutrinos. This requires, however, introduction of the non-standard interactions of neutrinos which lead to non-diagonal potentials in the flavor basis and therefore produce mixing.

In oscillations we test the dispersion relations, that is, the relations between the energy and momentum, and not masses immediately. Oscillations are induced because of difference of

dispersion of neutrino components that compose a mixed state. In vacuum the relation reads

$$E = \sqrt{p^2 + m^2} \approx p + \frac{m^2}{2p}. \quad (28)$$

The mass squared enters here (so, the chirality flips twice), and eventually there is no change of chirality: $\nu_L \rightarrow \nu_R \rightarrow \nu_L$. Therefore V, A interaction with medium can reproduce effect of mass.

In the presence of matter, the dispersion relation becomes

$$E = \sqrt{p^2 + m^2} + V \approx p + \frac{m^2}{2p} + V. \quad (29)$$

The matter potential can be considered as an effective mass. Indeed, for massless neutrinos Eq. (29) can be rewritten as $E = \sqrt{p^2 + m_{eff}^2}$, where

$$m_{eff}^2 = 2pV. \quad (30)$$

The effective mass has momentum dependence which allows to disentangle it from true mass.

It is consistency of results of many experiments in wide energy ranges and different environments: vacuum, matter with different density profiles that makes explanation of data without mass almost impossible.

In this connection one may wonder which type of experiment/measurement can uniquely identify the true mass? Let us mention three possibilities:

- Kinematical measurements: distortion of the beta decay spectrum near the end point. Notice that similar effect can be produced if a degenerate sea of neutrinos exists which blocks neutrino emission near the end point.
- Detection of neutrinoless double beta decay which is the test of the Majorana neutrino mass. Here complications are related to possible contributions to the decay from new L -violating interactions.
- Cosmology is sensitive to the sum of neutrino masses, and in future it will be sensitive to even individual masses. Here the problem is with degeneracy of neutrino mass and cosmological parameters.

V. IN CONCLUSION

In some cases (for historical or other reasons) terminology does not correspond to real physics. In most of the cases we understand difference and what is behind. Still bad terminology can be misleading producing wrong physics interpretations.

Calling the two different effects (oscillations and adiabatic conversion) just oscillations is simpler and shorter. In fact, both the oscillations and adiabatic conversion can be consequences of neutrino mass and mixing. Also neutrino decay is a consequence of mass and mixing, but we do not call it oscillations. Partly it was our fault with Mikheyev: In our early publications we described two different matter effects under the same name:

- Resonance enhancement of oscillations which takes place in matter with constant (quasi constant) density, like mantle of the Earth. Here the phase is crucial. Graphic representation of this effect is given in Fig. 11c. The effect hopefully will be observed by PINGU, ORCA experiments, and will allow us to determine the neutrino mass hierarchy.
- Adiabatic conversion of neutrinos which (as we have discussed) takes place in matter with slowly changing density (the Sun, supernovae). Resonance is important also here determining strength of transitions: strong transitions occur in the resonance channel when e.g. neutrinos are produced at densities much above the resonance one, cross the resonance layer and then exit matter at densities much below the resonance density.

In January 1986 at the Moriond workshop A. Messiah (he gave the talk [16]) asked me: “why do you call effect that happens in the Sun the resonance oscillations? It has nothing to do with oscillations, I will call it the MSW effect”. My reply was “yes, I agree, we simply did know how to call it. I will explain and correct this in my future talks and publications”. Messiah’s answer was surprising: “No way..., now this confusion will stay forever”. That time I could not believe him. I have published series of papers, delivered review talks, lectures in which I was trying to explain, fix terminology, *etc.* All this has been described in details in the talk at Nobel symposium [17], and for recent review see [8].

Ideally terminology should reflect and follow our understanding of the subject. Deeper understanding may require a change or modification of terminology. At the same time changing terminology is very delicate thing and can be done with great care.

In conclusion, the answer to the question in the title of the paper is

“Solar neutrinos: Almost No-oscillations”.

The SNO experiment has discovered effect of *the adiabatic flavor conversion* (the MSW effect). Oscillations (effect of the phase) are irrelevant. Evolution of the solar neutrinos can be considered as independent (incoherent) propagation of the produced eigenstates in matter. Flavors of these eigenstates (described by mixing angle) change according to density change. At high energies (SNO) the adiabatic conversion is close to the non-oscillatory transition which corresponds to production of single eigenstate. Oscillations with small depth occur in the matter of the Earth.

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APPENDIX

Here we provide some details for statements made in the text.

A. The amplitudes of the wave packets are proportional to the corresponding mixing parameters in Eq. (1). In ν_μ we have $g_2 = \cos \theta_{23}g$, $g_3 = \sin \theta_{23}g$, where g is the shape factor without mixing. Since the admixture of ν_τ in ν_2 is given by $\sin \theta_{23}$, the absolute value of amplitude of ν_τ from ν_2 equals $(\sin \theta_{23} \cos \theta_{23}g)$. Similarly, since the admixture of ν_τ in ν_3 is

$\cos \theta_{23}$, we obtain the absolute value of amplitude of ν_τ from ν_3 ($\sin \theta_{23} \cos \theta_{23} g$). The two amplitudes are equal. Consequently, in the initial moment when the oscillation phase is zero the two amplitudes cancel each other completely. So, the probability to find ν_τ : $P_{\mu\tau}(\phi_{osc}) = 0$. In the moment of time when $\phi_{osc} = \pi$ tau parts from two WP interfere constructively leading to the total amplitude $2 \sin \theta_{23} \cos \theta_{23} = \sin 2\theta_{23}$. Therefore the total probability to find ν_τ equals $P_{\mu\tau}(\phi_{osc} = \pi) = \sin^2 2\theta_{23}$. This determines the depth of oscillations. Dependence of the probability on the phase can be reconstructed using the above results for $\phi_{osc} = 0$ and π :

$$P_{\mu\tau}(\phi_{osc}) = \sin^2 2\theta_{23} \frac{1}{2} (1 - \cos \phi_{osc}) = \sin^2 2\theta_{23} \sin^2 \frac{\phi_{osc}}{2}.$$

This leads to the result in Eq. (9).

B. Eigenstates of the Hamiltonian and mixing: introduction of mixing in matter make sense once we can introduce these eigenstates. This is clearly possible for constant density. If density changes, the Hamiltonian depends on time, $H(t)$, so one can speak about the eigenstates of instantaneous Hamiltonian. The instantaneous eigenstates have sense if mixing (density) changes slowly enough. In this case the adiabatic regime is realized. Also one can compute corrections to the adiabatic results. If density changes quickly, so that corrections are large and adiabatic perturbation theory is broken, introduction of these eigenstates has no sense. In the case of constant or adiabatically changing density introduction of the eigenstates is useful for solution of the problem.

C. The adiabaticity condition is the condition under which the transitions $\nu_{1m} \leftrightarrow \nu_{2m}$ can be neglected. It has very simple expression in the resonance, where, in fact, it is most important. The width of the resonance layer Δr_R (the layer where $\sin^2 2\theta_{12}^m > 1/2$ and the angle changes from $\pi/8$ to $3\pi/8$) should be larger than the oscillation length in matter in the resonance l_m^R :

$$\Delta r_R \sim \tan 2\theta_{12} \left(\frac{\Delta n}{n dx} \right)^{-1} > l_m^R,$$

$l_m^R = l_\nu / \sin 2\theta_{12}$. Under this condition the system has enough time to adjust itself to changes of external conditions (matter density).

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